Constraints on Primordial Black Hole abundance from Stochastic Gravitational-Wave Background

— Coalescing events & Curvature perturbations

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SW, T. Terada, K. Kohri, arXiv:1903.05924 [astro-ph.CO]

SW, Y.-F. Wang, Q.-G. Huang, T.G.F. Li , Phys. Rev. Lett. 120, 191102 (2018)

@Wuhan; April 28, 2019

Who am I ?

- 2018/08—Now: PD; Institute of Particle and Nuclear Studies @ KEK
- 2016/06—2018/08: PD; The Chinese University of Hong Kong
- 2014/06—2016/06: PD; Institute of Theoretical Physics @ CAS

2009/09—2014/06: PhD; Institute of High Energy Physics @ CAS

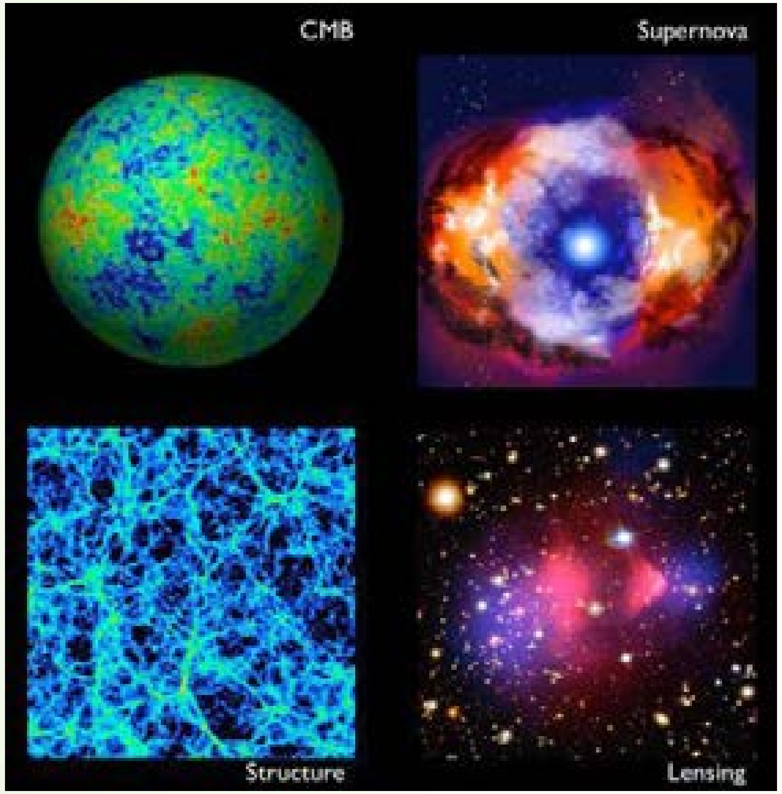
2005/09—2009/06: BSc; Huazhong University of Science and Technology

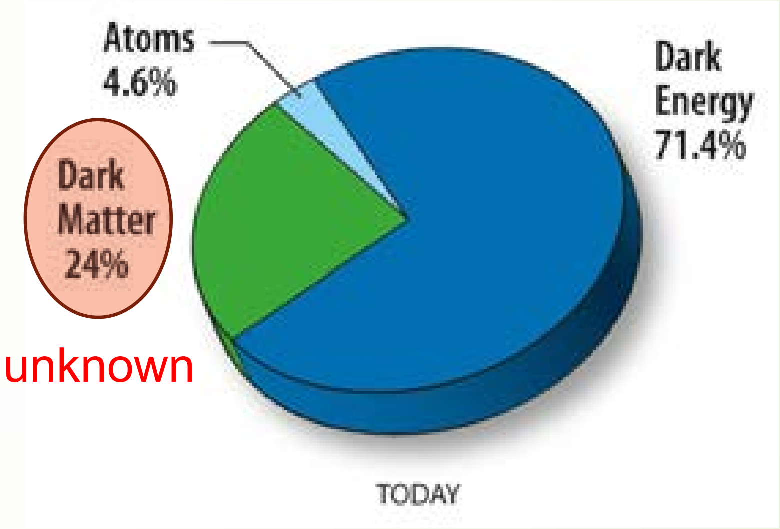
Research: the early Universe, dark matter/energy… by CMB, GWs, 21cm lines…

Outline

- Introduction & Motivations
- PBHs can account for aLIGO's event rate
- New observational window: SGWBs
  - Coalescing events
  - Curvature perturbations
- Summary

Dark matter (DM)

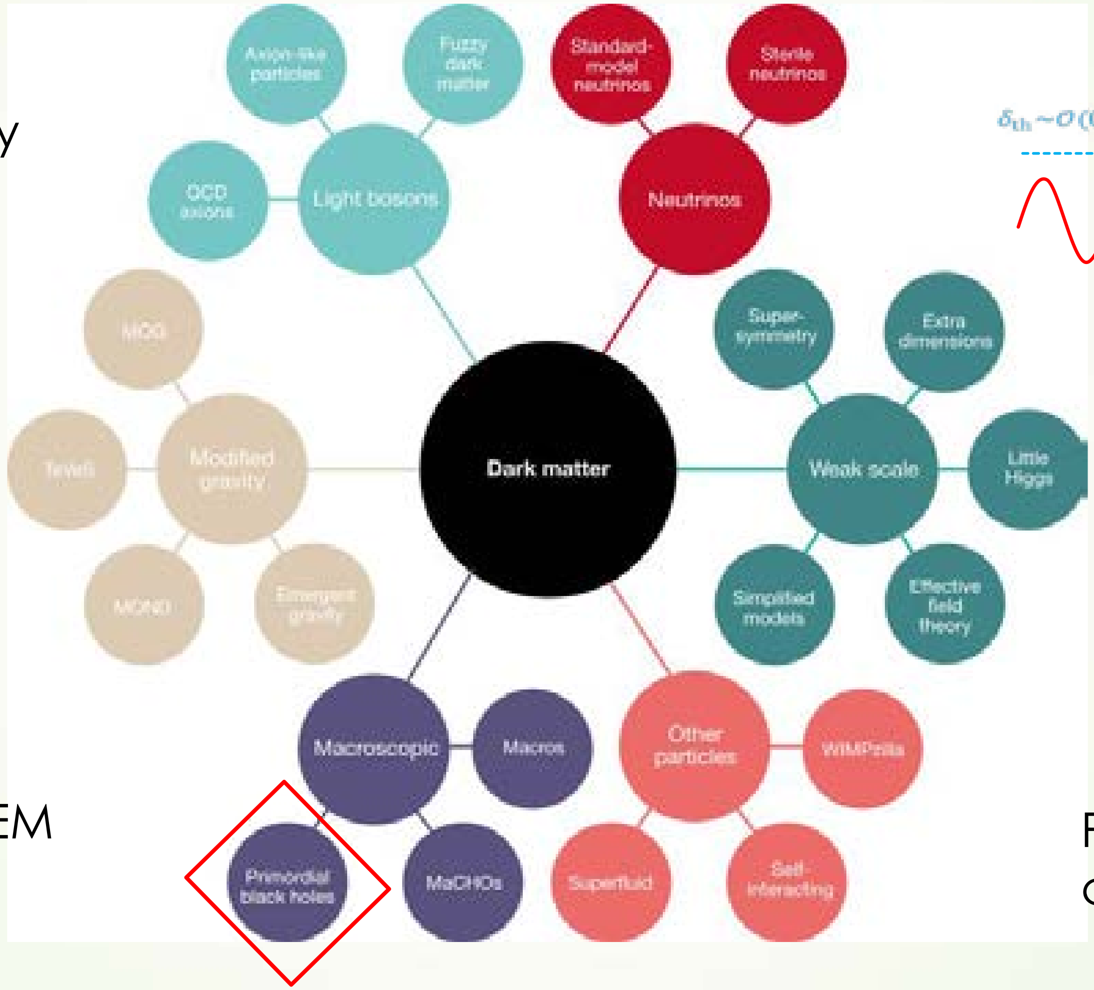


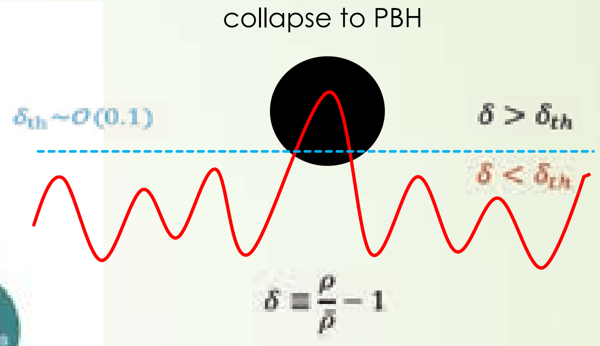


The energy density of DM is more than
5 times that of visible matter !

Primordial Black Holes (PBHs) as DM candidate

PBHs were proposed by S. Hawking





"DARK": no EM radiation, invisible

PBHs are one of the candidates for DM

Primordial power spectrum on small scales

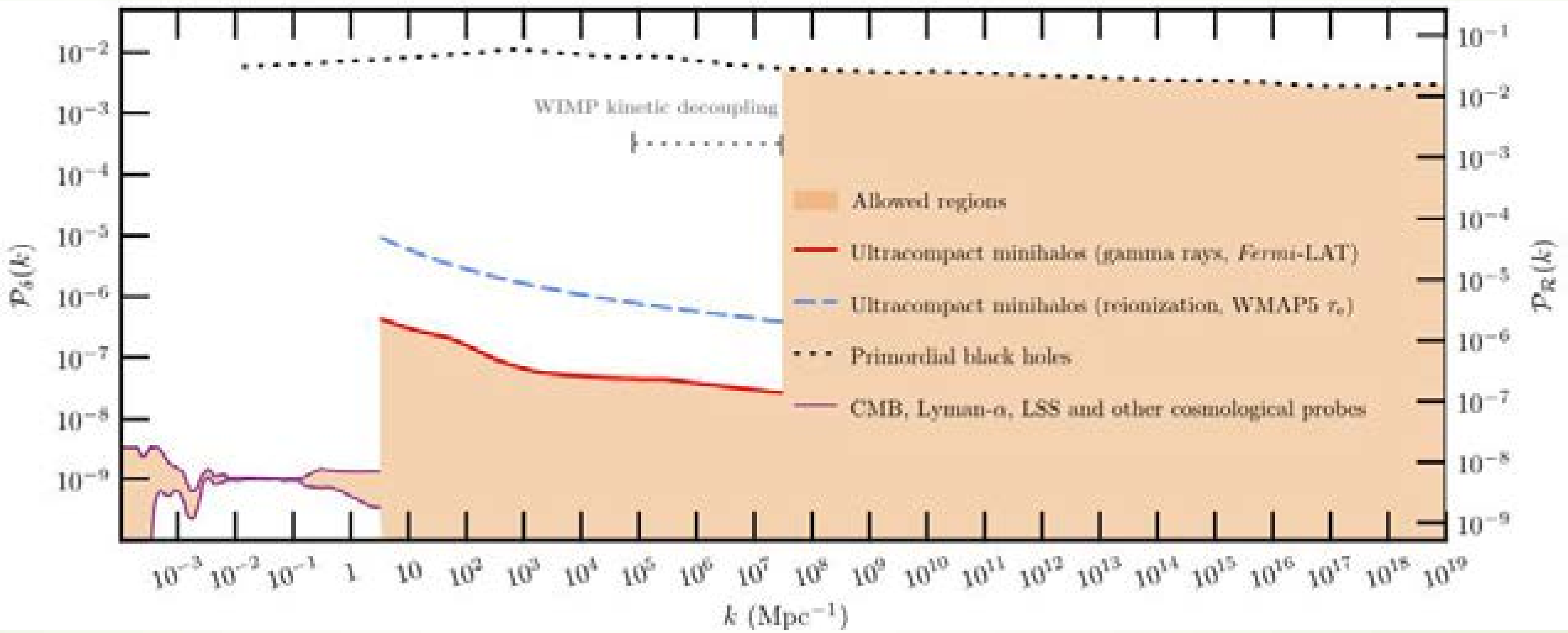
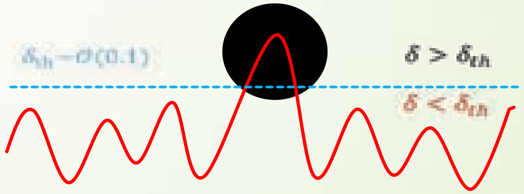
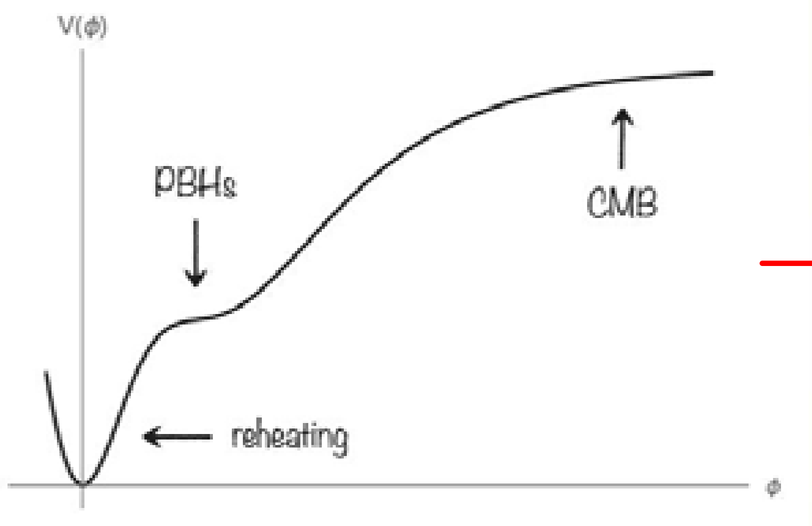
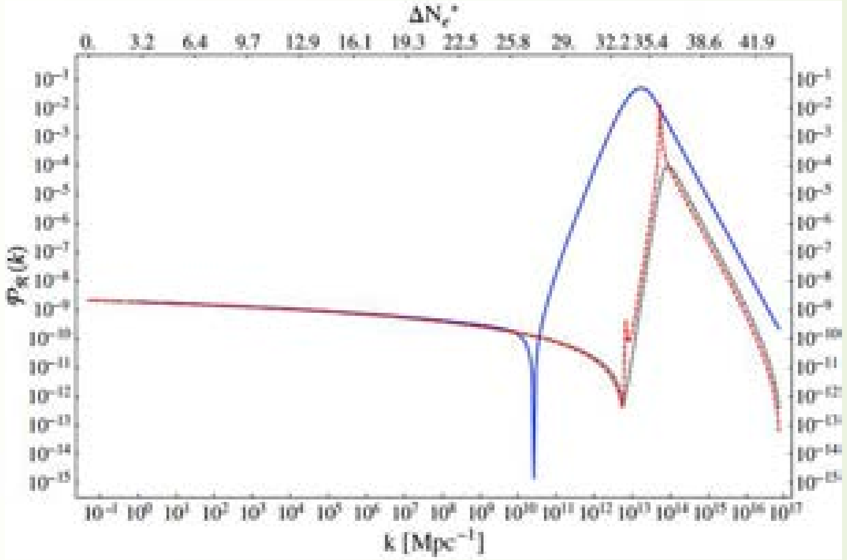


Figure from Bringmann, Scott, Akrami, Phys. Rev. D 85, 125027 (2012)



PBHs are important for studying Inflaton

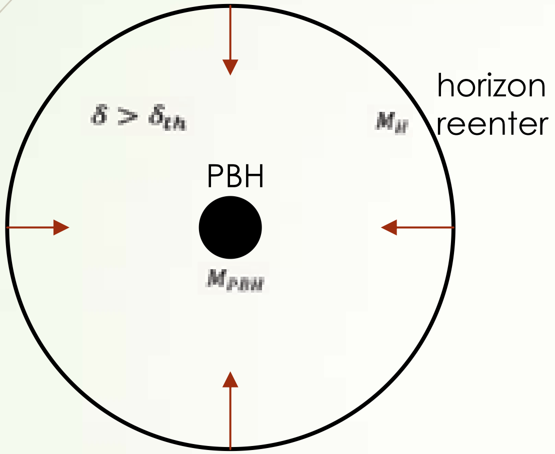




$$A_s = \frac{1}{24\pi^2 M_P^2}\frac{V}{\epsilon_V}, \quad \text{where } \epsilon_V = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2$$

Figures from Ballesteros, Taoso, Phys. Rev. D 97, 023501 (2018)

PBHs: Formation & Abundance



$$M_{PBH} = \alpha M_H (\delta - \delta_{th})^{\gamma}$$

$$\alpha = 3.3, \gamma = 0.36, \delta_{th} = 0.45$$

- Possibility of PBH production (Press-Schechter formula)
- Assume Gaussian PDF:

$$P_{M_H}(\delta(M_{PBH})) \propto \exp\left(-\frac{\delta^2(M_{PBH})}{2\sigma^2(M_H)}\right)$$

- Given scalar power spectrum P_ζ, then

dilution due to radiation dominated

- Mass function of PBHs (in DM):
- Abundance of PBHs:

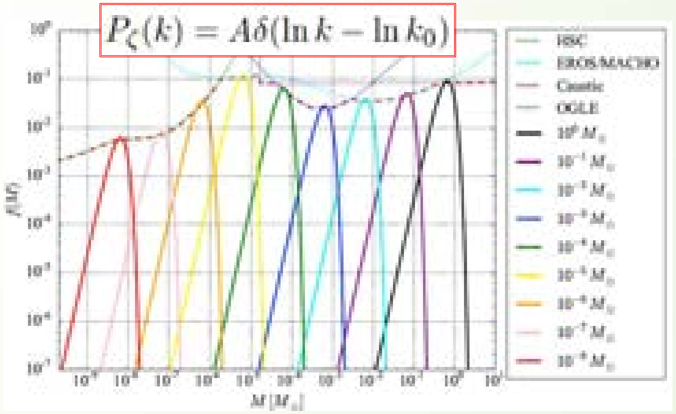
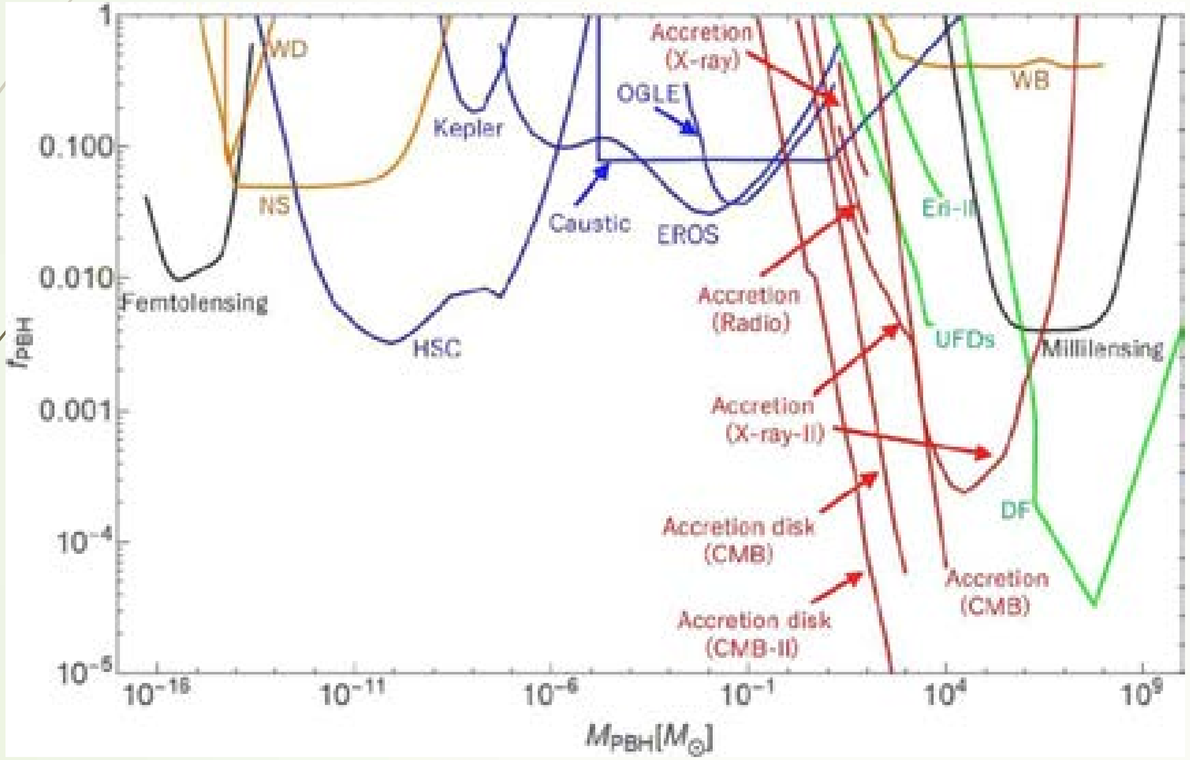
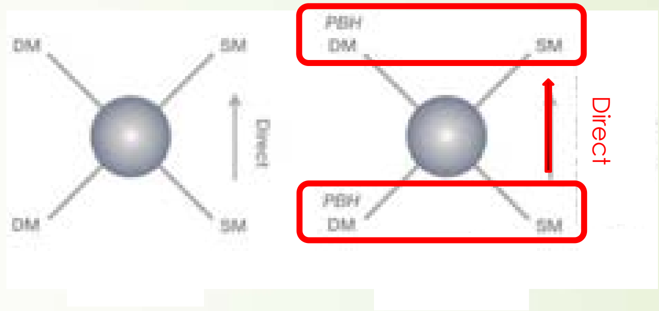


Figure from SW, T. Terada, K. Kohri, arXiv:1903.05924

Direct constraints on PBH abundance

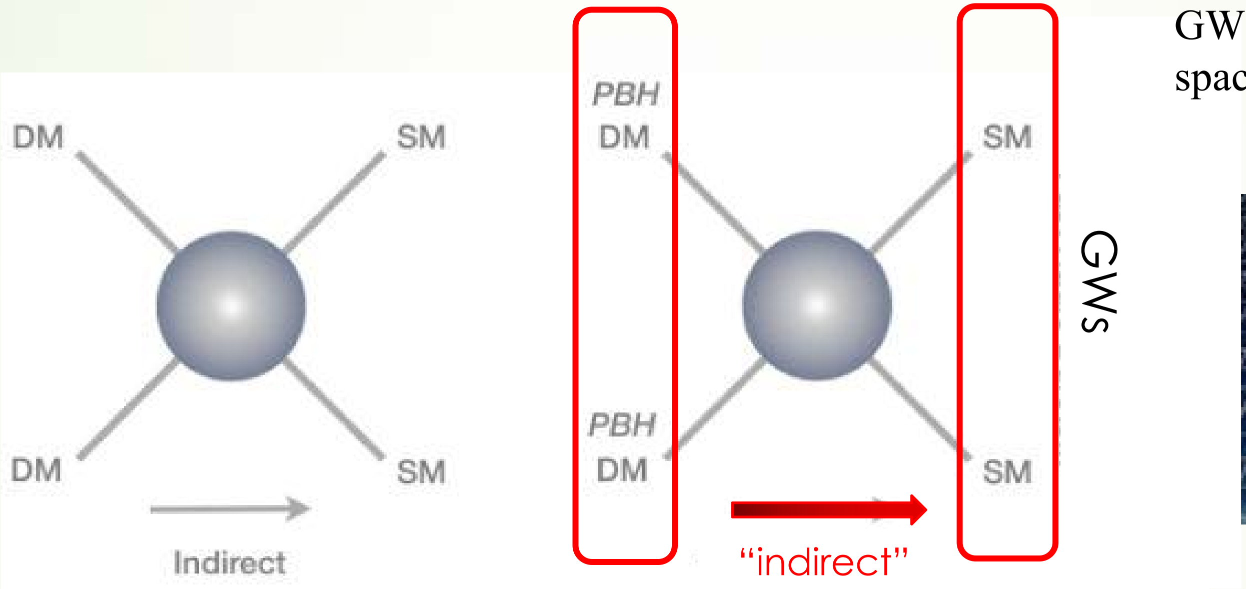




The direct detection looks for the effects of PBHs on standard astrophysical objects.
(analogy with direct detection of DM particles)

“Indirect” ones: **G**ravitational **W**aves (GWs)

This is just an analogy with indirect constraints on DM.
In fact, GWs provide (most?) DIRECT constraints.



The “indirect” detection looks for GWs emitted when a PBH binary coalesces to a larger BH.

GWs are ripples in the curvature of spacetime fabric (Einstein, 1916)

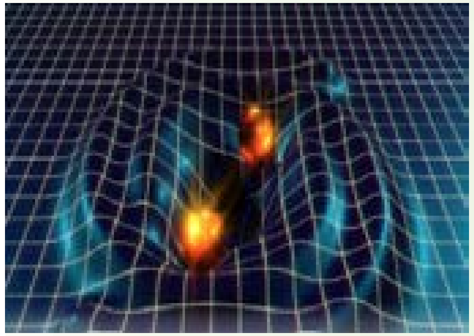
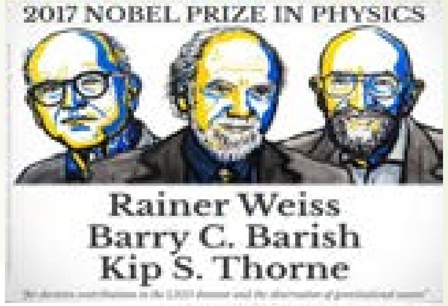


Figure: GWs are generated by two compact objects orbiting each other

GW150914: First GWs from BBH merger

- **B**inary **B**lack **H**oles (BBHs) indeed exist
- They can merge within the age of the Universe



PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_{\odot}$ and $29^{+4}_{-4} M_{\odot}$, and the final black hole mass is $62^{+4}_{-4} M_{\odot}$, with $3.0^{+0.5}_{-0.5} M_{\odot} c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

Are PBHs observed by Advanced LIGO (aLIGO) ?

Event	$m_1/M_\odot$	$m_2/M_\odot$	$\mathcal{M}/M_\odot$	$\chi_{\rm eff}$	$M_f/M_\odot$	a_f	$E_{\rm rad}/(M_\odot c^2)$	$\ell_{\rm peak}/({\rm erg\,s^{-1}})$	$d_L/{\rm Mpc}$	z	$\Delta\Omega/{\rm deg}^2$
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.6}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.3}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$3.1^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4}\times10^{56}$	430^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	180
GW151012	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.0}_{-1.1}$	$0.04^{+0.28}_{-0.19}$	$35.7^{+9.9}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	$1.5^{+0.5}_{-0.5}$	$3.2^{+0.8}_{-1.7}\times10^{56}$	1060^{+540}_{-480}	$0.21^{+0.09}_{-0.09}$	1555
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$8.9^{+0.3}_{-0.3}$	$0.18^{+0.20}_{-0.12}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	$1.0^{+0.1}_{-0.2}$	$3.4^{+0.7}_{-1.7}\times10^{56}$	440^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$	1033
GW170104	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$21.5^{+2.1}_{-1.7}$	$-0.04^{+0.17}_{-0.20}$	$49.1^{+5.2}_{-3.9}$	$0.66^{+0.08}_{-0.10}$	$2.2^{+0.5}_{-0.5}$	$3.3^{+0.6}_{-0.9}\times10^{56}$	960^{+430}_{-410}	$0.19^{+0.07}_{-0.08}$	924
GW170608	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$7.9^{+0.2}_{-0.2}$	$0.03^{+0.19}_{-0.07}$	$17.8^{+3.2}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	$0.9^{+0.05}_{-0.1}$	$3.5^{+0.4}_{-1.3}\times10^{56}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$	396
GW170729	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$35.7^{+6.5}_{-4.7}$	$0.36^{+0.21}_{-0.25}$	$80.3^{+14.6}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	$4.8^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-1.5}\times10^{56}$	2750^{+1350}_{-1320}	$0.48^{+0.19}_{-0.20}$	1033
GW170809	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$25.0^{+2.1}_{-1.6}$	$0.07^{+0.16}_{-0.16}$	$56.4^{+5.2}_{-3.7}$	$0.70^{+0.08}_{-0.09}$	$2.7^{+0.6}_{-0.6}$	$3.5^{+0.6}_{-0.9}\times10^{56}$	990^{+320}_{-380}	$0.20^{+0.05}_{-0.07}$	340
GW170814	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$24.2^{+1.4}_{-1.1}$	$0.07^{+0.12}_{-0.11}$	$53.4^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	$2.7^{+0.4}_{-0.3}$	$3.7^{+0.4}_{-0.5}\times10^{56}$	580^{+160}_{-210}	$0.12^{+0.03}_{-0.04}$	87
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$	$0.00^{+0.02}_{-0.01}$	≤ 2.8	≤ 0.89	≥ 0.04	$\geq 0.1\times10^{56}$	40^{+10}_{-10}	$0.01^{+0.00}_{-0.00}$	16
GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$26.7^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$	$59.8^{+4.8}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	$2.7^{+0.5}_{-0.5}$	$3.4^{+0.5}_{-0.7}\times10^{56}$	1020^{+430}_{-360}	$0.20^{+0.07}_{-0.07}$	39
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$29.3^{+4.2}_{-3.2}$	$0.08^{+0.20}_{-0.22}$	$65.6^{+9.4}_{-6.6}$	$0.71^{+0.08}_{-0.10}$	$3.3^{+0.9}_{-0.8}$	$3.6^{+0.6}_{-0.9}\times10^{56}$	1850^{+840}_{-840}	$0.34^{+0.13}_{-0.14}$	1651

We cannot determine whether they arise from astrophysical process or primordial collapse.
Both are possible. But, at least, PBHs can account for the local merger rate observed.

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- Introduction & Motivations
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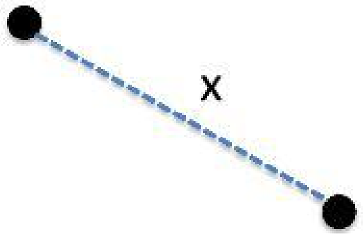
Formation of PBH binaries

Nakamura, 1997; Sasaki, 2016

- The physical mean separation of PBHs at matter-radiation equality is

$$\bar{x} = \frac{1}{1+z_{\rm eq}} \left(\frac{M_{\rm pbh}}{f_{\rm pbh}\Omega_{\rm CDM}\rho_{\rm crit}} \right)^{1/3}$$



- The pair of PBHs is supposed to decouple from the expansion of the Universe and forms a gravitational bound system if the average energy density of PBHs over the volume is larger than the background cosmic energy density.

$$R_m = \frac{a_m}{a_{\rm eq}} = \frac{1}{f}\frac{x^3}{\bar{x}^3} \quad (< 1)$$

$$\rho_{\rm crit} = \frac{3H_0^2}{8\pi G} = 2.775 \times 10^{20} h^2 \frac{M_\odot}{\rm Gpc^3}$$

- They just coalesce to a single black hole on the free fall time scale if the motion of these two PBHs is not disturbed.

$$t_f = (R_m x)^{3/2}/(GM_{\rm pbh})^{1/2}$$

(x is comoving distance at $z=z_{eq}$)

Formation of PBH binaries

Nakamura, 1997; Sasaki, 2016

- The tidal force from neighboring black holes provides enough angular momentum to keep the black holes from colliding with each other.

- The major axis is

$$a = R_m x = x^4/(f\bar{x}^3)$$

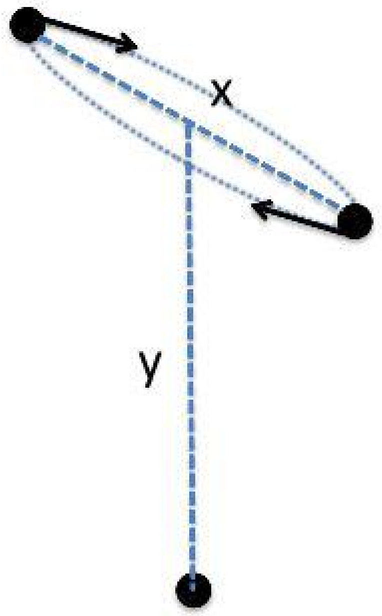
- The minor axis is

$$b = (\text{tidal acceleration}) \times t_f^2$$

$$= GM_{\rm pbh}\frac{R_m x}{(R_m y)^3} \times t_f^2 = \left(\frac{x}{y}\right)^3 a$$

- The eccentricity of the binary at the formation time are

$$e = \sqrt{1 - \left(\frac{x}{y}\right)^6} \le e_{max} = \sqrt{1 - f^{\frac{3}{2}}\left(\frac{a}{\bar{x}}\right)^{\frac{3}{2}}}$$



(x, y are comoving distances at z=z$_{eq}$

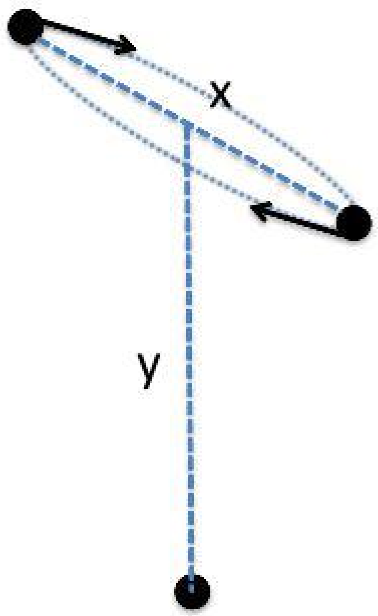
Formation of PBH binaries

Nakamura, 1997; Sasaki, 2016

✧ The coalescence time for the PBH binary due to gradually shrink by gravitational radiations is (Peters and Mathews, 1963; Peters, 1964)

$$t = Qa^4(1 - e^2)^{7/2}$$

$$Q = \frac{3}{170}(GM_{\rm pbh})^{-3}$$



◊ Assuming uniform PDF both for x and y in 3D space, one can calculate the probability that two PBHs would coalesce

$$dP = \frac{9}{\bar{x}^3}x^2y^2dxdy$$

which can be transformed in terms of e and t

(x, y are comoving distances at $z=z_{eq}$)

Account for aLIGO's local event rate

Nakamura, 1997; Sasaki, 2016

- The probability that the coalescence occurs within the time interval $(t, t+dt)$ is given by

$$dP_t = \begin{cases} \frac{3}{58}\left[-\left(\frac{t}{T}\right)^{\frac{3}{8}} + \left(\frac{t}{T}\right)^{\frac{3}{37}}\right]\frac{dt}{t}, & \text{for } t < t_c \\ \frac{3}{58}\left(\frac{t}{T}\right)^{\frac{3}{8}}\left[-1 + \left(\frac{t}{t_c}\right)^{-\frac{29}{56}} f^{-\frac{29}{8}}\right]\frac{dt}{t}, & \text{for } t \geq t_c \end{cases}$$

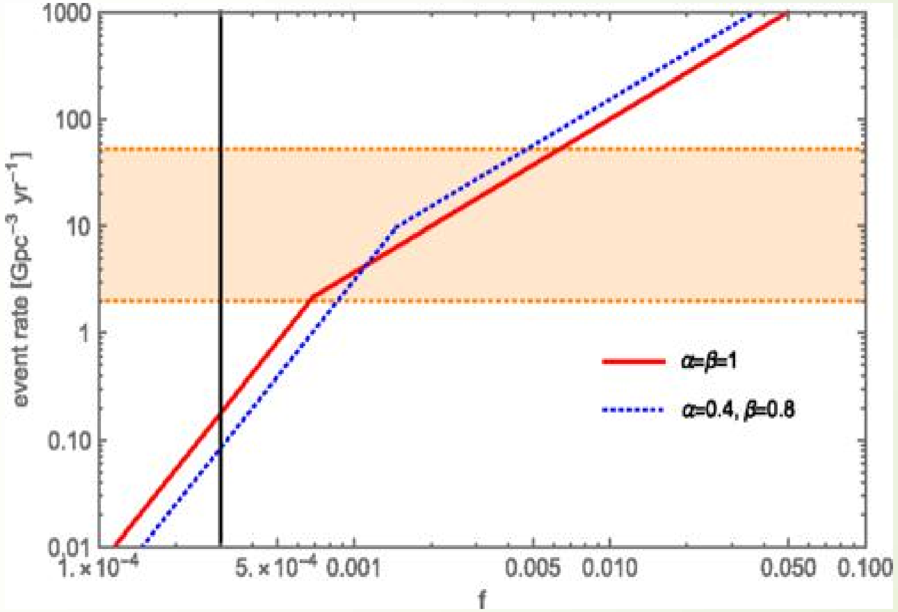
where $T = \frac{3}{170}\frac{c^5 \bar{x}^4}{(GM_{\rm PBH})^3 f^4}$ and $t_c = \frac{3}{170}\frac{c^5 \bar{x}^4 f^{25/3}}{(GM_{\rm PBH})^3}$

- The merger rate of PBH binaries is defined by [Sasaki etal, 2016]

$$R_{\rm PBH}(z) = \frac{3H_0^2}{8\pi G}\frac{f\Omega_{\rm DM}}{M_{\rm PBH}}\frac{dP_t}{dt}$$

where z is related to cosmic time $t = t_0 - \frac{1}{H_0}\int_0^z \frac{dz'}{(1+z')E(z')}$, t_0 is the age of the Universe today, and $E(z) = H(z)/H_0$ is the reduced Hubble parameter

$f \sim \mathcal{O}(0.001)$ can account for aLIGO's local merger rate.



O1, O2 constraints on (subsolar-mass) PBH abundance

- aLIGO directly search the sub-solar mass ($0.2M_{\odot} \sim 1.0M_{\odot}$) ultra-compact binaries in O1, O2

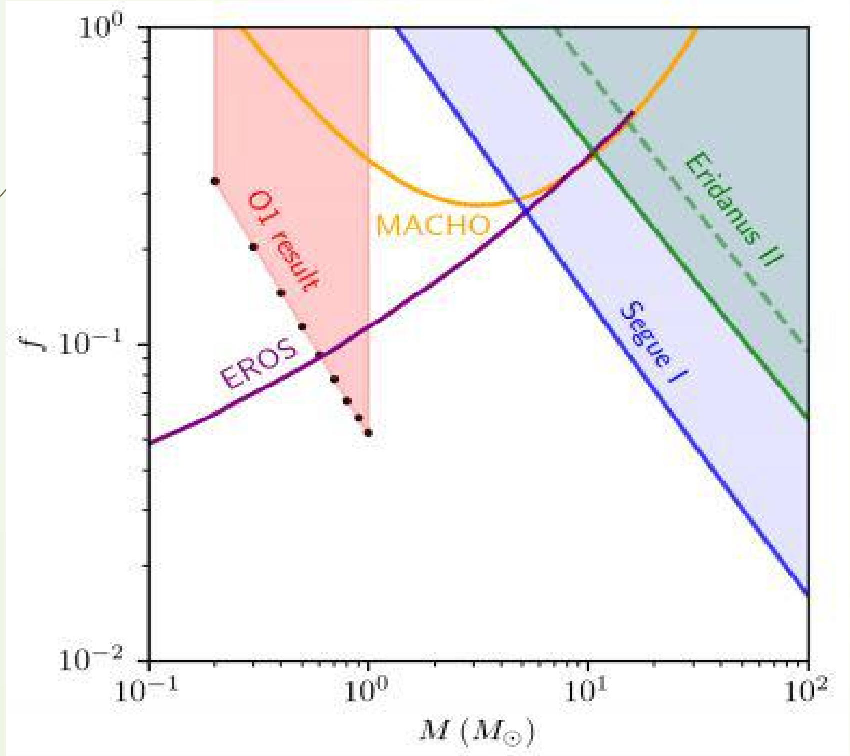


Figure from LIGO arXiv:1808.04771

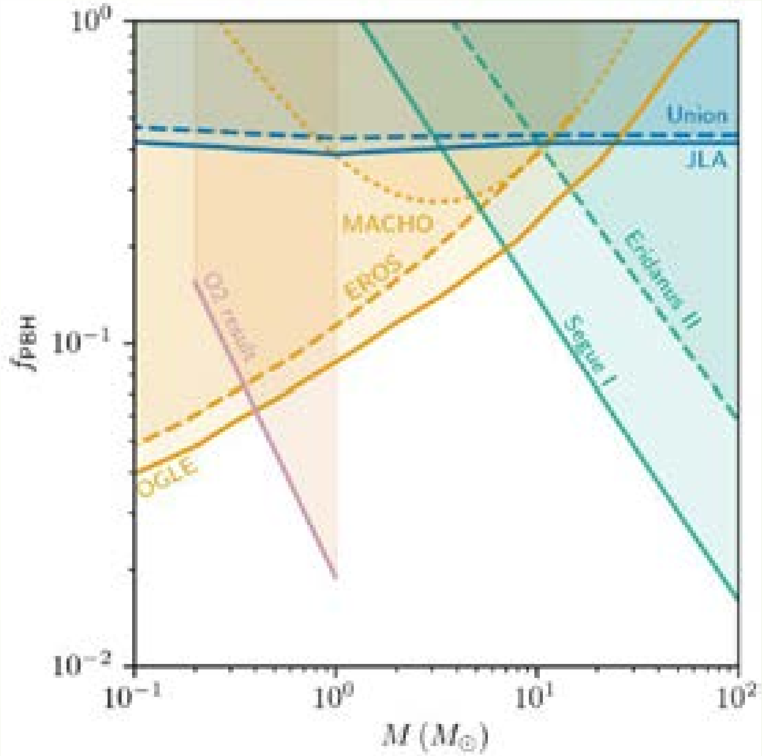


Figure from LIGO arXiv:1904.08976

Challenges

GW strain: $h \propto M_c^{5/6}$ where $M_c = (m_1 m_2)^{3/5} (m_1 + m_2)^{-1/5}$

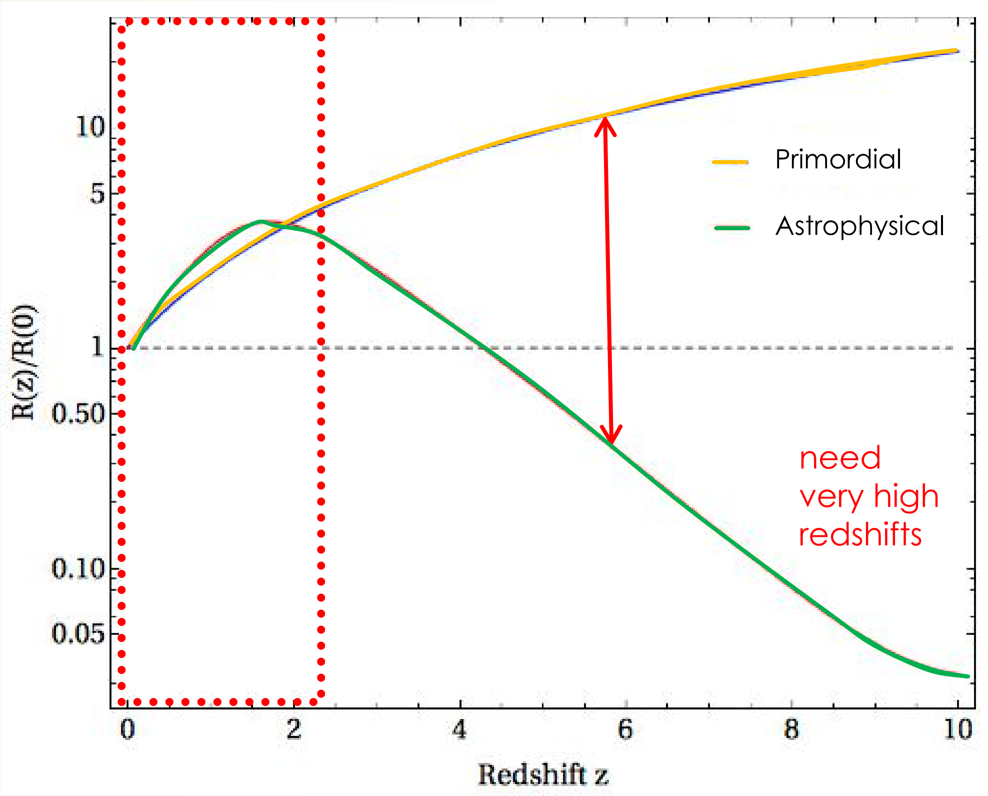
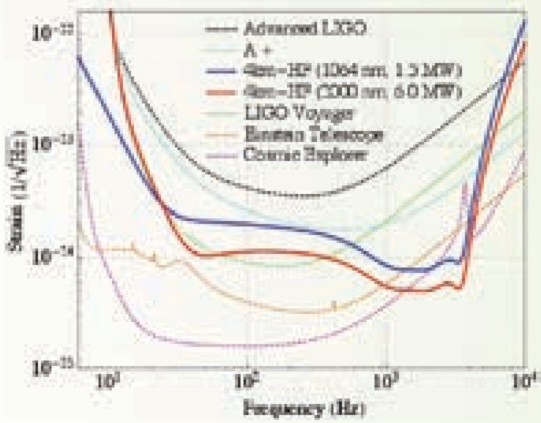


Figure from Z. Chen, Q.-G. Huang, arXiv:1904.02396



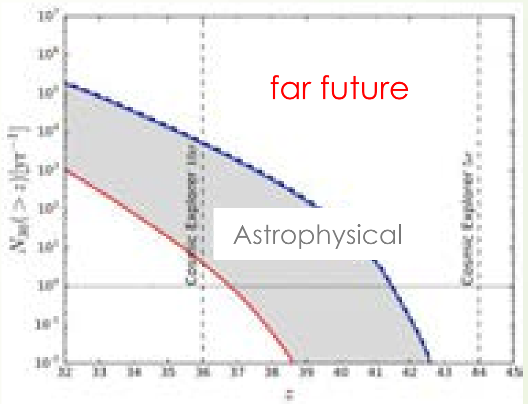


Figure from S. Koushiappas, A. Loeb, Phys.Rev.Lett. 119 (2017) no.22, 221104

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Stochastic Gravitational Wave Background (SGWB)

Conventions from M. Maggiore, Phys.Rept. 331 (2000) 283

- Metric: $ds^2 = -dt^2 + \left(\delta_{ij} + \frac{1}{2}h_{ij}\right)dx^i dx^j$

  where h_{ij} is GW=tensor perturbation

- Energy density: $\rho_{GW} = \frac{1}{32\pi G}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$

  spatial average of 00-component of energy - momentum tensor over several wavelengths

- Energy density spectrum: $\Omega_{GW}(\nu) = \frac{8\pi G}{3H_0^2}\frac{d\rho_{GW}}{d\ln\nu}$

  where ν is GW frequency; H_0 is Hubble constant

- Two types of SGWB:
  - from coalescing events of PBH binaries
  - from curvature perturbations associated with production of PBHs

SGWB from coalescing events

arXiv:1904.08976

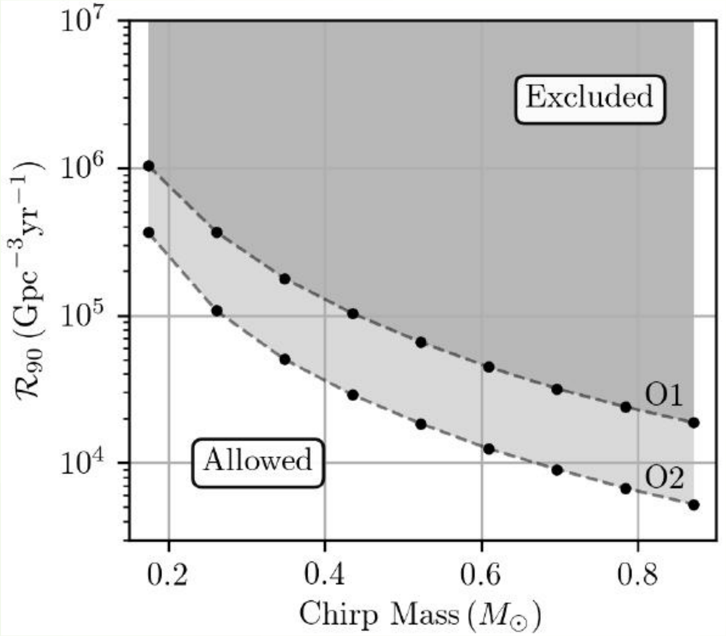
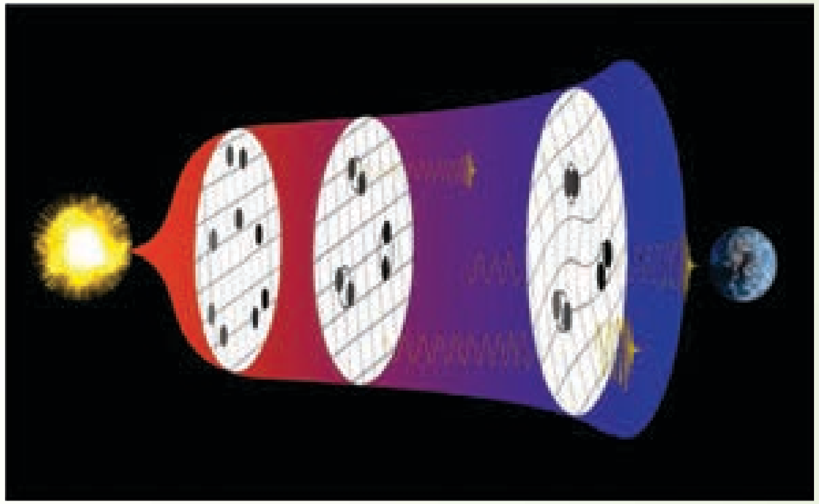


Figure by SW, Y.-F. Wang, Q.-G. Huang, T.G.F. Li

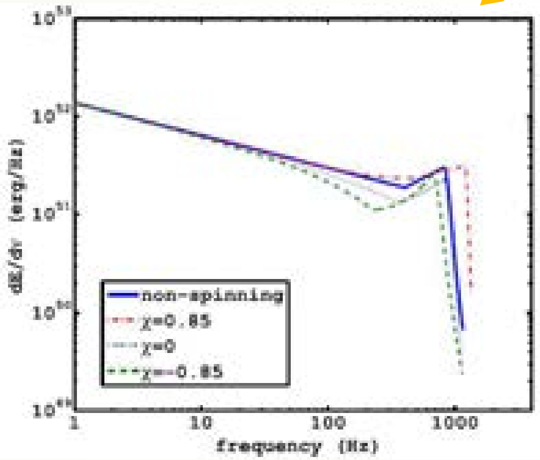


This SGWB originates from an incoherent super-position of GWs from all the un-resolved PBH mergers in the Universe.

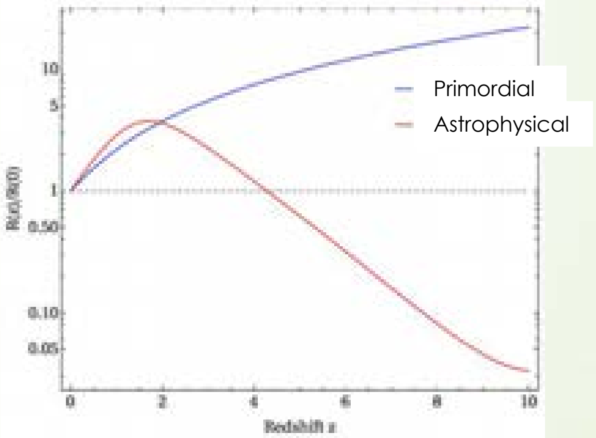
Energy density spectrum of the SGWB from coalescence events

SW, Y.-F. Wang, Q.-G. Huang, T.G.F. Li ,
Phys. Rev. Lett. 120, 191102 (2018)

$$\Omega_{\rm GW}(\nu; M_{\rm PBH}, f) = \frac{\nu}{\rho_c H_0} \int_0^{z_{\rm sup}} \frac{R_{\rm PBH}(z; M_{\rm PBH}, f)}{(1+z)E(z)} \times \frac{dE_{\rm GW}}{d\nu_s}(\nu_s) dz,$$



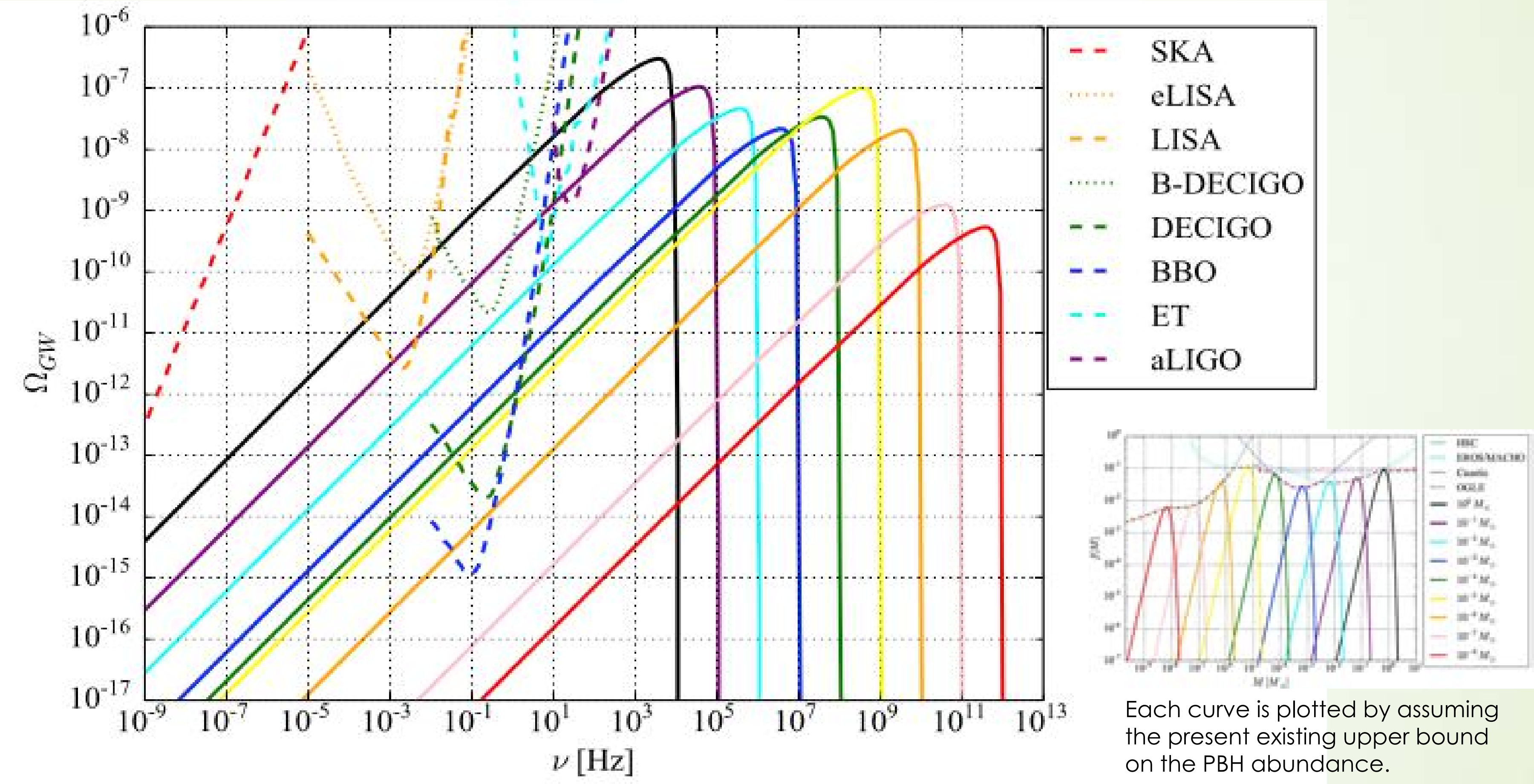
GW energy density spectrum for $10M_\odot$—$10M_\odot$ BBH IMR (Ajith, 2011)



Normalized merger rate in terms of the redshift z (Sasaki, 2016)

Sensitivity curves as upper bounds

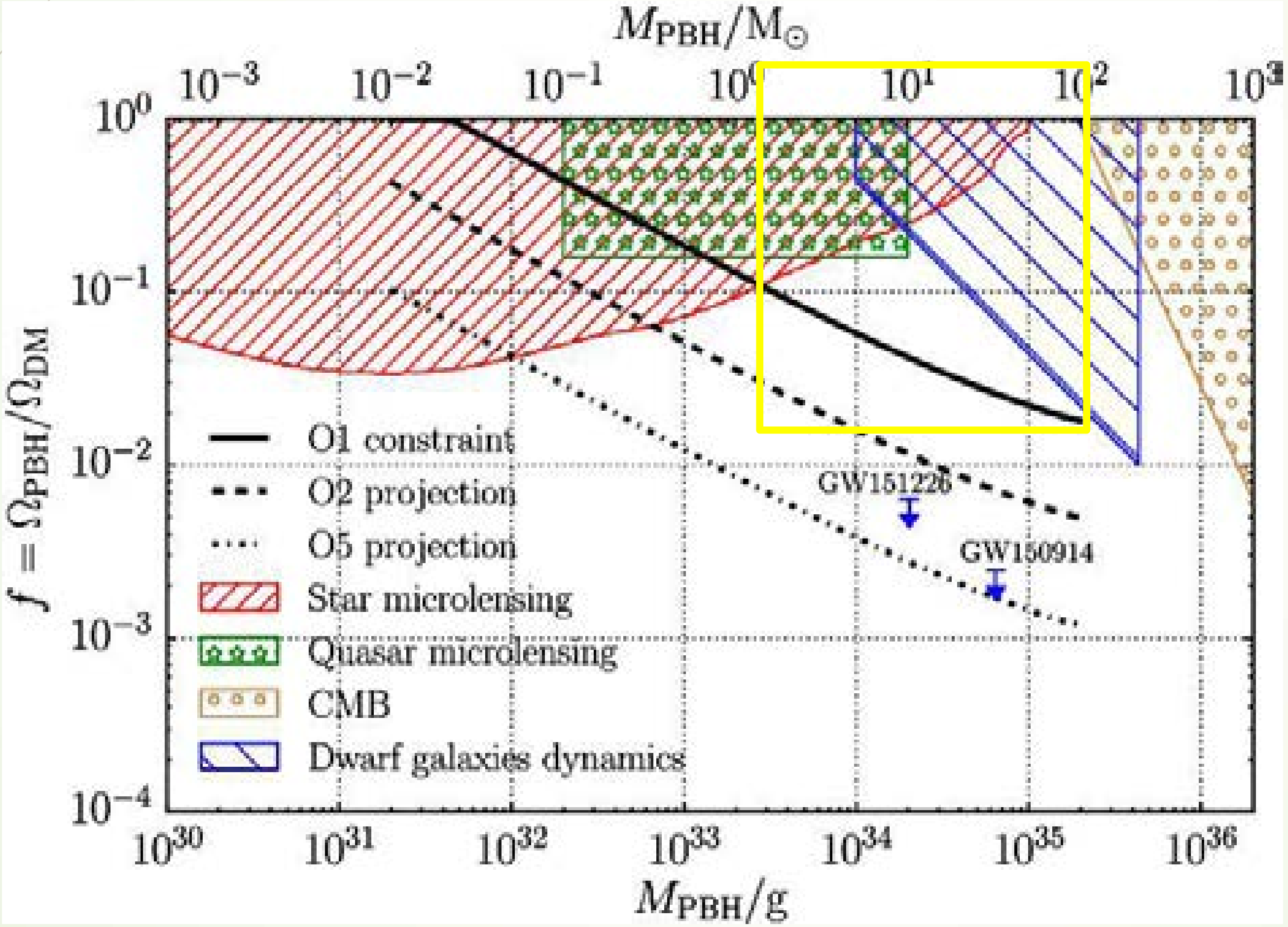
SW, T. Terada, K. Kohri, arXiv:1903.05924

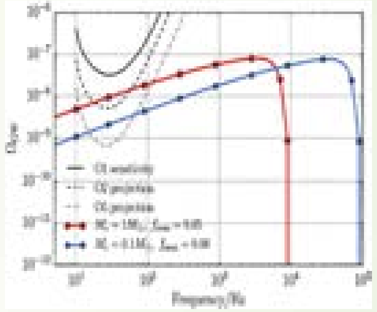


Each curve is plotted by assuming the present existing upper bound on the PBH abundance.

aLIGO O1 constraint on PBH abundance

SW, Y.-F. Wang, Q.-G. Huang, T.G.F. Li , Phys. Rev. Lett. 120, 191102 (2018)

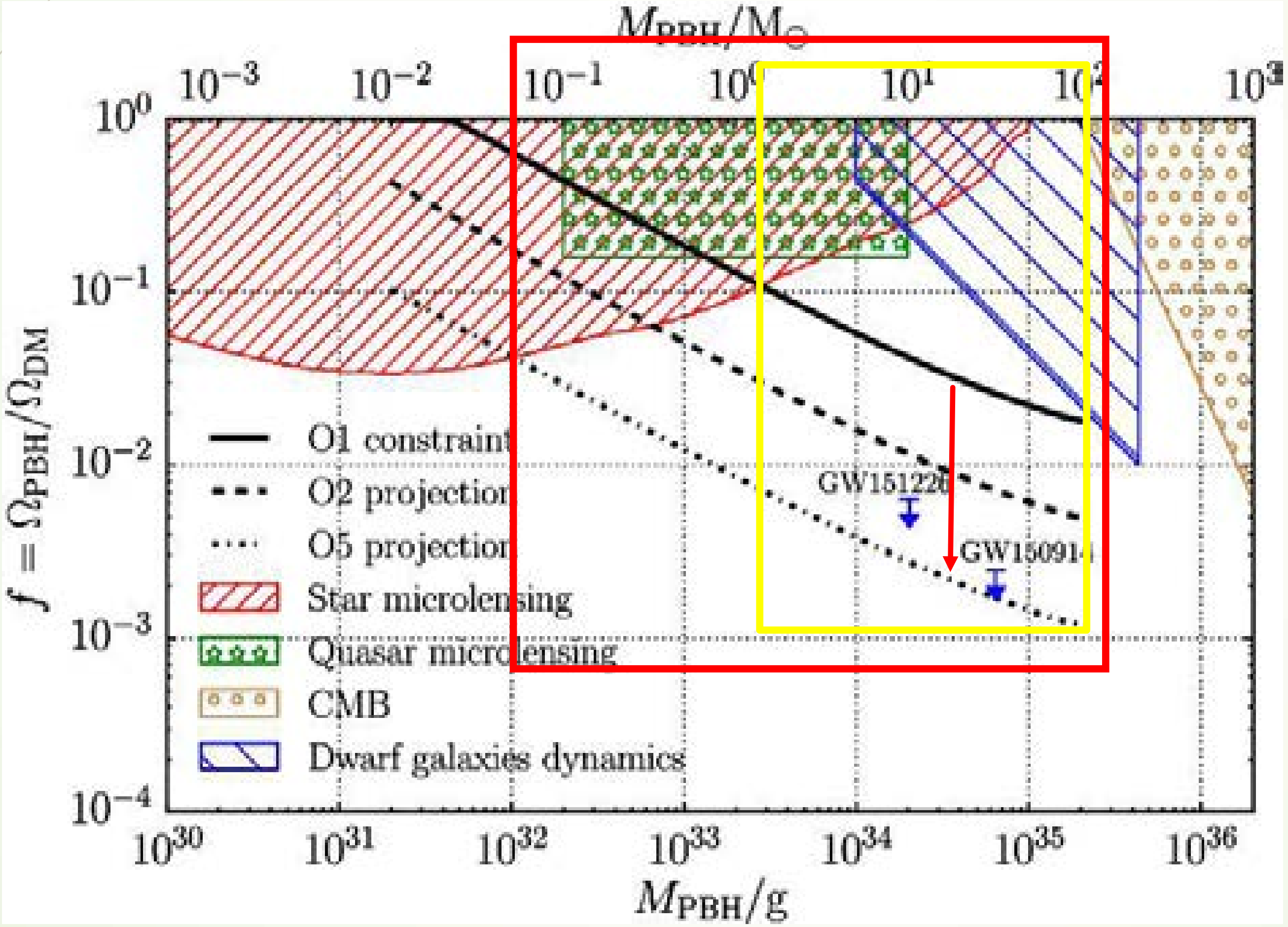


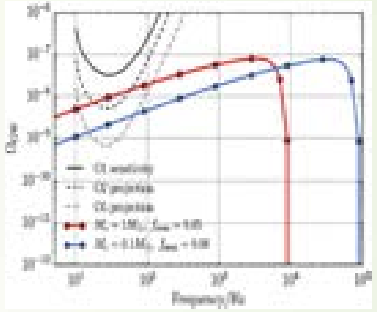


- aLIGO O1 provides the tightest bound on f, which is less than $\mathcal{O}(10^{-1})$, for PBH masses of $(1\sim10^2)\ M_\odot$
- This constraint improves the existing ones by about one order of magnitude

aLIGO future constraints on PBH abundance

SW, Y.-F. Wang, Q.-G. Huang, T.G.F. Li , Phys. Rev. Lett. 120, 191102 (2018)

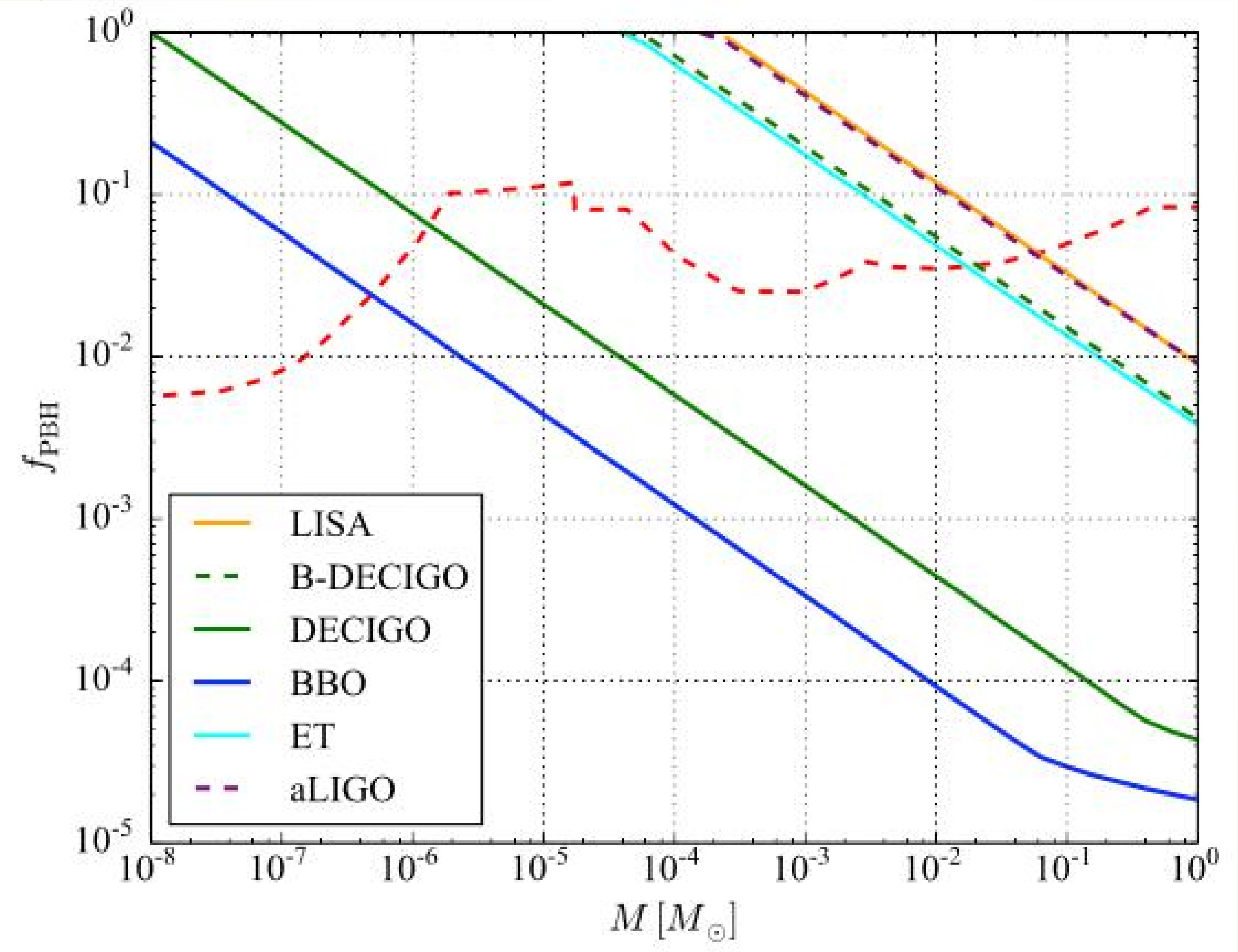


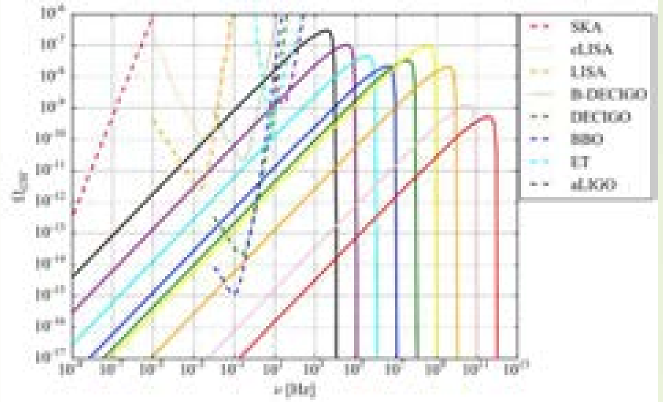


- aLIGO O5 will further improve O1 by about one order of magnitude
- It will also provide the tightest upper limits for wider mass range: $\mathcal{O}(10^{-1}$-$10^{2})$ $M_\odot$

For other GW detectors

SW, T. Terada, K. Kohri, arXiv:1903.05924

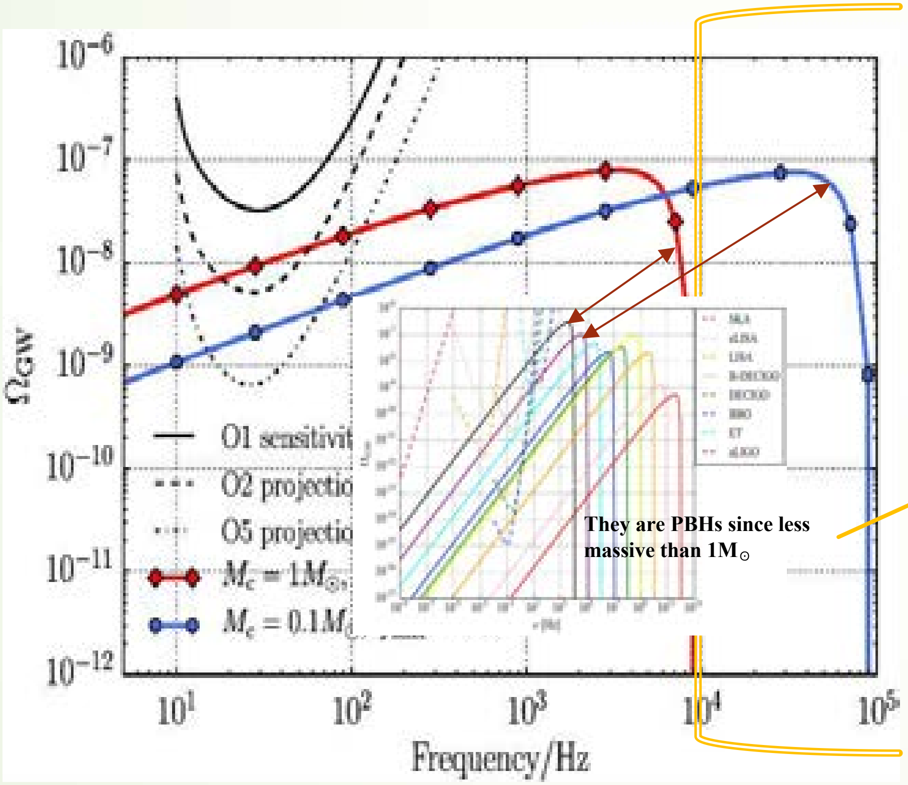


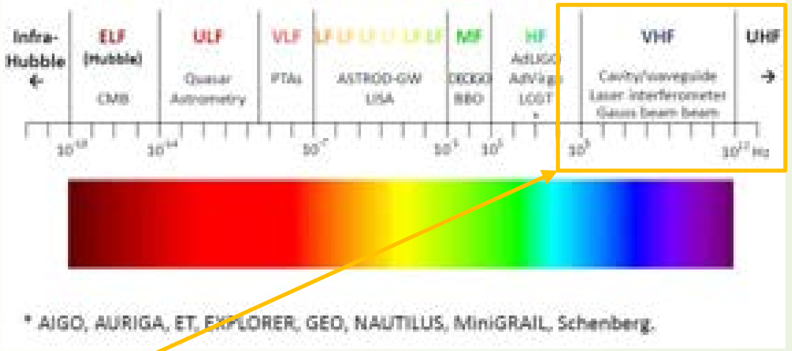


- Other detectors will improve aLIGO upper bounds in the future
- In particular, DECIGO and BBO will improve by at least two orders of magnitude for even wider mass range: $\mathcal{O}(10^{-6}\text{-}10^0)\ M_\odot$

SGWB from small-mass BHs: “smoking gun” of PBHs

SW, Y.-F. Wang, Q.-G. Huang, T.G.F. Li , Phys. Rev. Lett. 120, 191102 (2018)





W.-T. Ni, arXiv:1003.3899

- Decisive evidence for PBHs need a detection of SGWB at **higher** frequency bands than (e.g.) aLIGO.

Outline

- Introduction & Motivations
- PBHs can account for aLIGO's event rate
- New observational window: SGWBs
  - Coalescing events
  - Curvature perturbations
- Summary

SGWB from curvature perturbations

The induced SGWB must exist due to the existence of curvature perturbations.

GW

- Perturbed metric: $ds^2 = a(\eta)^2 \left[-(1-2\Phi)\, d\eta^2 + \left(1 + 2\Phi + \frac{1}{2} h_{ij}\right) dx^i dx^j \right]$
- Equation of motion: $h''_{\mathbf{k}}(\eta) + 2\mathcal{H} h'_{\mathbf{k}}(\eta) + k^2 h_{\mathbf{k}}(\eta) = 4 S_{\mathbf{k}}(\eta)$
- Source term:

$$\mathcal{S}(\mathbf{k},\eta) = 36 \int \frac{d^3 l}{(2\pi)^{3/2}} \frac{l^2}{\sqrt{2}} \sin^2\theta \begin{pmatrix} \cos 2\varphi \\ \sin 2\varphi \end{pmatrix} \Phi_{\mathbf{l}} \Phi_{\mathbf{k}-\mathbf{l}} \times \left[j_0(ux) j_0(vx) - 2\frac{j_1(ux) j_0(vx)}{ux} - 2\frac{j_0(ux) j_1(vx)}{vx} + 3\frac{j_1(ux) j_1(vx)}{uvx^2} \right].$$

nonlinear

$\sim \frac{4}{9} P_\zeta$

where $u = |\mathbf{k}-\mathbf{l}|/k$

$v = l/k$

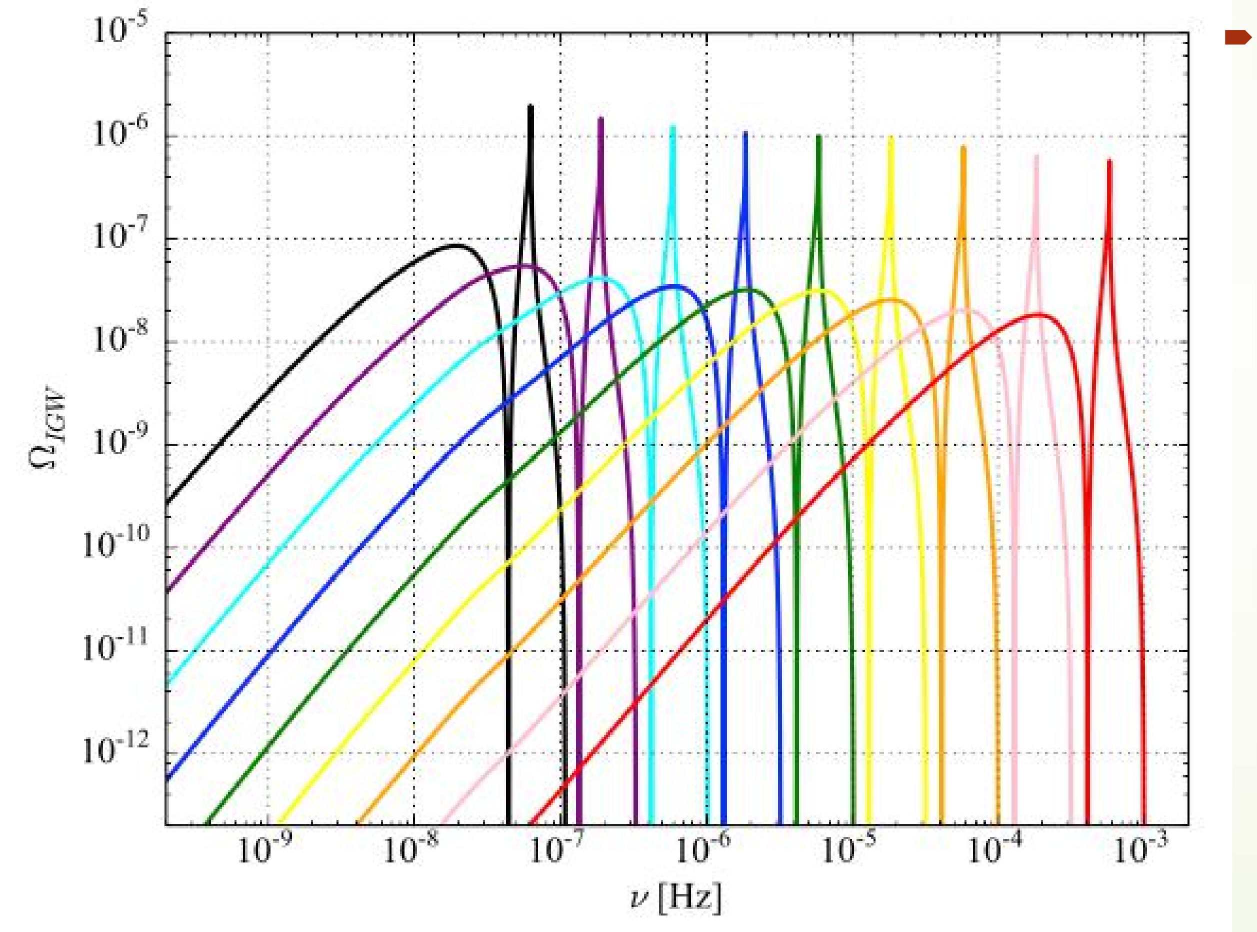
$x = k\eta/\sqrt{3}$

- SGWB spectrum: $\Omega_{\rm GW}(\eta,k) \longrightarrow \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)}$

$$\langle h^{\lambda}_{\mathbf{k}}(\eta) h^{\lambda'}_{\mathbf{k}'}(\eta) \rangle = \delta_{\lambda\lambda'} \delta^3(\mathbf{k}+\mathbf{k}') \frac{2\pi^2}{k^3} \mathcal{P}_h(\eta,k)$$

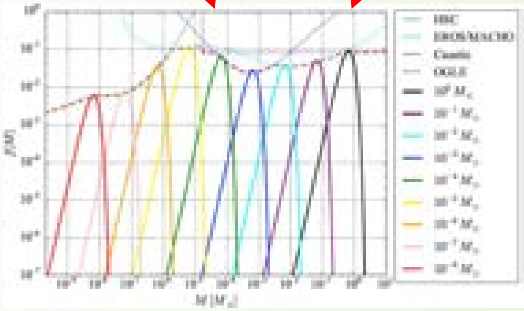
overline denotes time average

Energy density spectrum of induced SGWB



- Consider the power spectrum of primordial curvature perturbations to be a delta function of ln*k* :

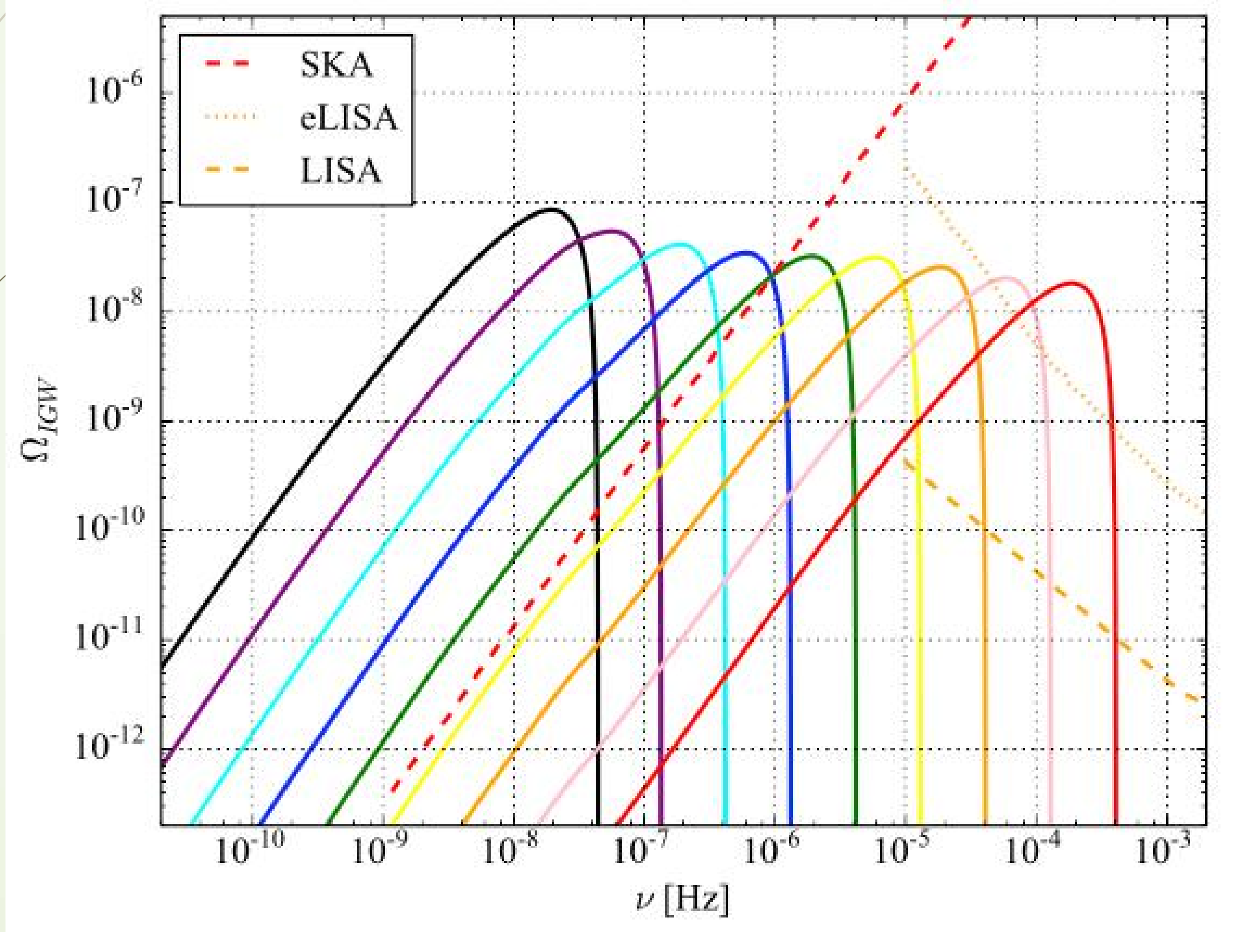
$$P_\zeta(k) = A\delta(\ln k - \ln k_0)$$

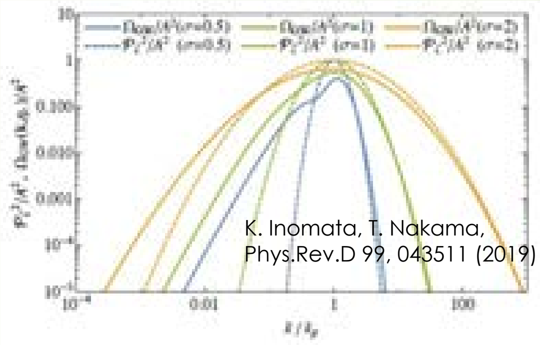


Each curve is plotted by assuming the present existing upper bound on the PBH abundance.

SW, T. Terada, K. Kohri, arXiv:1903.05924

Compare with experimental sensitivities





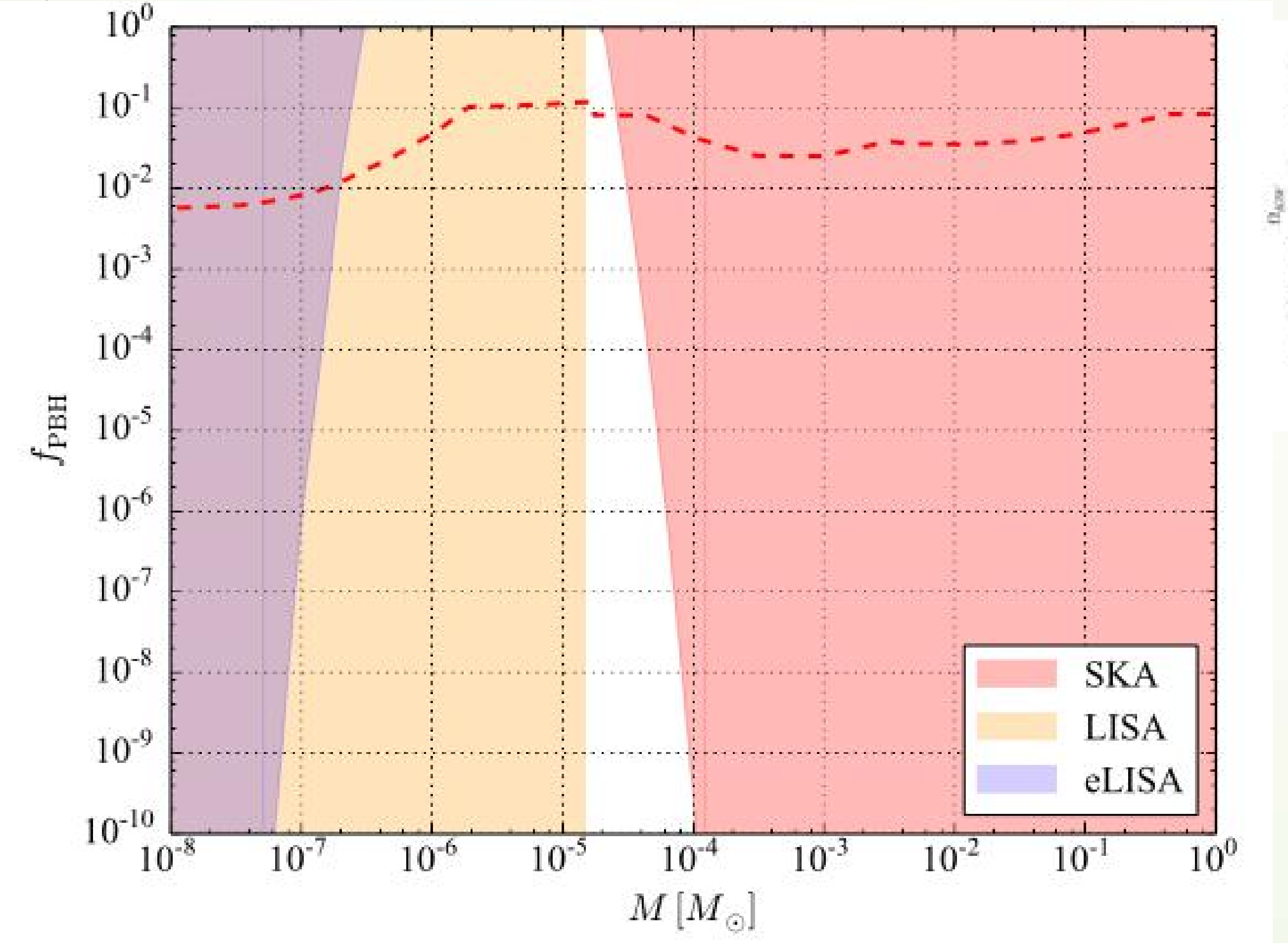
K. Inomata, T. Nakama, Phys.Rev.D 99, 043511 (2019)

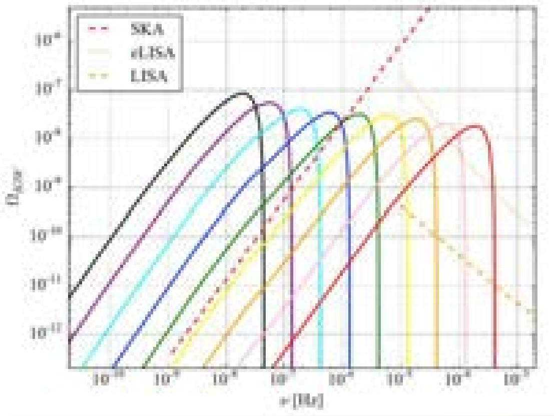
$$\mathcal{P}_\zeta(k) = A \exp\left(-\frac{(\log k - \log k_p)^2}{2\sigma^2}\right)$$

- We only consider the most conservative upper bounds in our work

SW, T. Terada, K. Kohri, arXiv:1903.05924

Expected constraints on PBH abundance

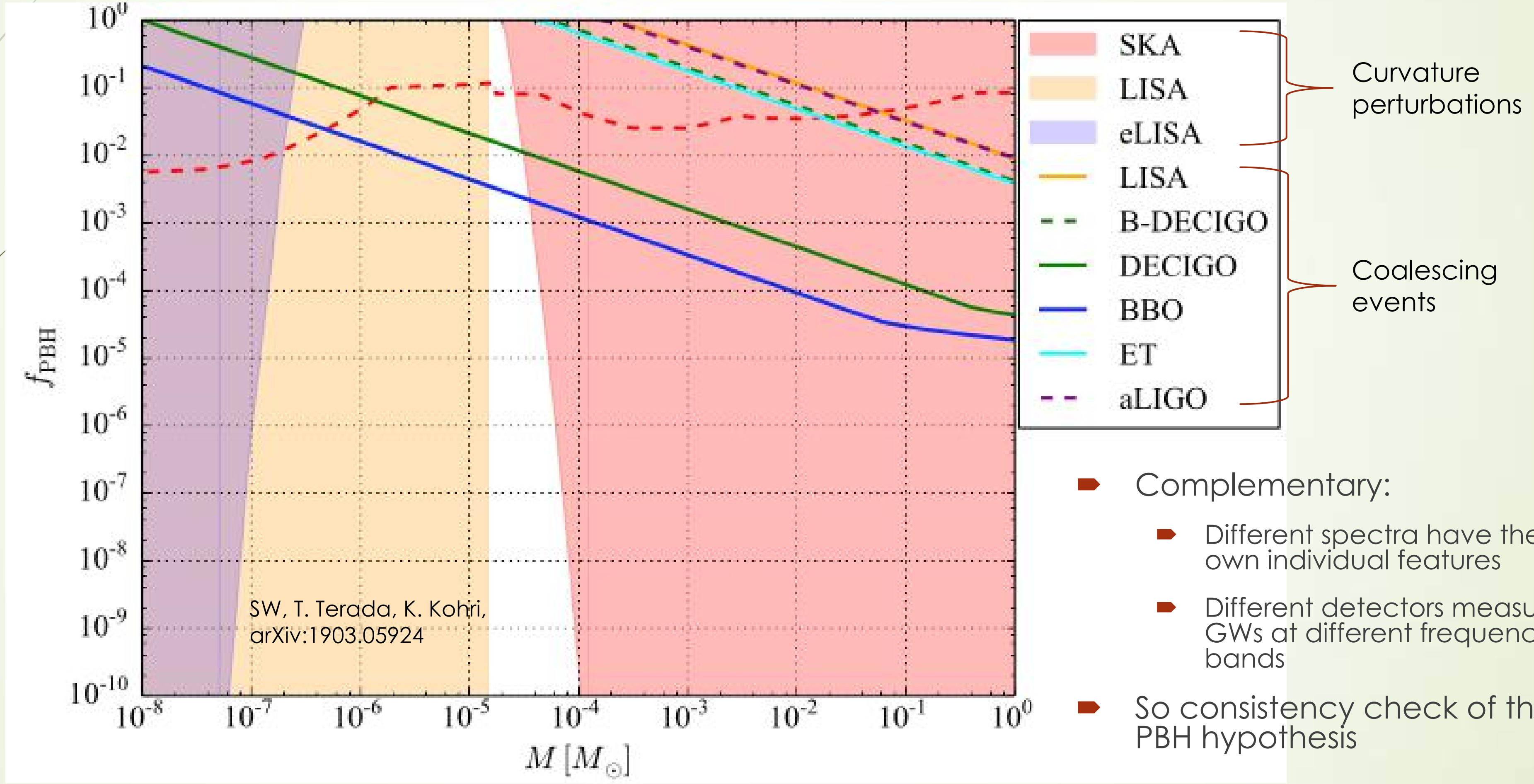




- the null detection of the induced SGWB could exclude the shaded regions

SW, T. Terada, K. Kohri, arXiv:1903.05924

Compare two types of SGWB constraints



- Complementary:
  - Different spectra have their own individual features
  - Different detectors measure GWs at different frequency bands
- So consistency check of the PBH hypothesis

Outline

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Summary

- Calculate the SGWB spectra from binary PBH coalescences
- Calculate the induced SGWB from enhanced curvature perturbations
- Forecast the constraints on PBH abundance from some GW detectors
- Both approaches are complementary and useful for testing the PBH hypothesis

Please feel free to cite our papers : ☺

- Sai Wang, Takahiro Terada, and Kazunori Kohri, "*Prospective constraints on the primordial black hole abundance from the stochastic gravitational-wave backgrounds produced by coalescing events and curvature perturbations*", under review by Phys. Rev. D, arXiv: 1903.05924 [astro-ph.CO].
- Sai Wang, Yi-Fan Wang, Qing-Guo Huang, and Tjonnie G. F. Li, "*Constraints on the Primordial Black Hole Abundance from the First Advanced LIGO Observation Run Using the Stochastic Gravitational-Wave Background*", Phys. Rev. Lett. 120 (2018) no.19, 191102.
- Email address: physics0911@163.com

Thanks very much for your kind attentions!